ROBUSTNESS OF TIME-VARYING DISCRETE PLANT SWITCHING CONTROL

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Abstract: This paper describes a supervised discrete-time switching control algorithm. The supervisor chooses the controller among a finite set of candidate controllers. From a set of candidate models the chosen controller is the one corresponding to the best fitted model. The process is assumed to be modelled by a linear time-varying discrete system which can be described by a system that commutes among a finite set of discrete linear time-invariant models. This paper proves the stability robustness of the closed loop system if the time interval between consecutive commuting instants is large enough.

Keywords: Switching control, Robustness, Time-Varying, Multiple Model,

1. INTRODUCTION

This paper describes and analyzes a supervised discrete-time switching control algorithm. The supervisor chooses the controller, among a set of candidate controllers, comparing a bank of shared-state estimators and choosing the controller that corresponds to the 'best fitted' model. Such control systems have been proposed in (Morse, 1995; Morse, 1996; Borrelli, 1999) considering linear time-invariant plants. The problem of the robustness with unmodelled dynamics have been studied under this approach in subsequent work (Morse, 1997). A stability analysis of switching controlled system is presented in (Rato, 2002) for a class of slowly time-varying discrete plants. A switching control algorithm based on hidden Markov models has also been proposed in (Lemos, 1999) without a stability analysis.

In this paper the the control scheme follows the one proposed in (Morse, 1996) and sufficient conditions are derived to ensure the stability robustness of the controlled system with a linear time-varying discrete plant. In section 2 the problem is formulated and the control algorithm is presented. In section 3 the main result is presented and used to prove the stability of the overall system. Simulation results are shown in section 4. Section 5 summarizes some conclusions.

2. PROBLEM FORMULATION

Consider the set of n admissible nominal linear plants $\{P_1, P_2, ..., P_n\}$ known *a priori* and controllers $\{K_1, K_2, ..., K_n\}$, and admit that process P commute among the elements of $\{P_1, P_2, ..., P_n\}$. Further admit, that the closed loop composed by $\{K_i, P_i\}$ is stable with margin at least λ_{CL} and at each instant k, the active controller $K_{\sigma(k)}$ is decided by a estimator based supervisor with output $\sigma(k)$, Figure 1.

Admitting the plant varies slowly enough (with a minimum interval T_{Dp} between changes), and the supervisor is defined as in (Rato, 1997; Borrelli, 1999), the problem of interest is to analyze the stability (in the BIBO sense) of the overall system composed by the controller, supervisor and plant in the presence of unmodelled dynamics. Figure 1 depicts a block diagram

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of the feedback control loop. The Plant dynamics is

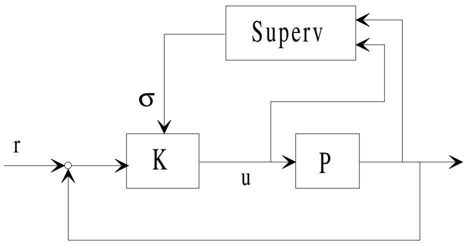

Fig. 1. Feedback loop with Controller, Plant and Supervisor.

defined by a nominal plant and unmodelled dynamics of multiplicative type. Thus, between changes the plant dynamics is defined by,

$$y = (1 + \delta_m)\frac{B}{A}u \, + \, d \tag{1}$$

where δ_m is a limited norm transfer function.

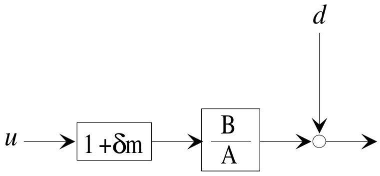

Fig. 2. Plant with multiplicative type unmodelled dynamics.

Figure 2 depicts a block diagram describing the plant dynamics between changes.

2.1 *Switching Control*

The plant is modelled by discrete linear time-invariant system P_i around each operating point defined by the occurrence of a given operating regime. This plant is assumed to be detectable and stabilizable. A linear local controller K_i is associated to each of the above local models. The mission of the supervisor is to compare the set of admissible plants with the true one and switch among controllers accordingly. Consider a supervisor constituted by a multiple estimator, a performance evaluator and a switching logic scheme. The estimator generates a set of estimates of the system output, the performance evaluator makes an average of the squared estimator errors, and the switching logic scheme commutes among controllers according to the performance signals assuring that each controller remains active at least T_D sampling instants, denoted as the *dwell-time*. A convenient way to construct the multiple estimator as well as the bank of controllers is in a shared-state form. This form, which is a non-minimal realization, has the advantage of concentrating the dependence of the parameters in the output equation. Thus, consider the shared-state multiple estimator defined by,

$$x_E(k+1) = \begin{bmatrix} A_E & 0 \\ 0 & A_E \end{bmatrix} x_E(k) +$$

$$\begin{bmatrix} b_E \\ 0 \end{bmatrix} y(k) + \begin{bmatrix} 0 \\ b_E \end{bmatrix} u(k) \tag{2}$$

$$\hat{y}_p(k) = C_p x_E(k) \tag{3}$$

where A_E has eigenvalues inside de circle of radius $\lambda_E < 1$; (A_E, b_E) is a controllable pair and C_p depends on the parameters of the $p - th$ plant model. Consider the shared-state multiple controller.

$$x_C(k+1) = \begin{bmatrix} A_C & 0 \\ 0 & A_C \end{bmatrix} x_C(k) +$$

$$\begin{bmatrix} b_C \\ 0 \end{bmatrix} (r(k) - y(k)) + \begin{bmatrix} 0 \\ b_C \end{bmatrix} u(k) \tag{4}$$

where A_C as eigenvalues inside the circle of radius $\lambda_C < 1$, and (A_C, b_C) is a controllable pair. The control signal is given by

$$u_\sigma(k) = f_\sigma x_C(k) + g_\sigma(r(k) - y(k)) \tag{5}$$

where f_σ and g_σ are vectors depending on the parameters of the σ-th local controller. Consider the performance evaluator defined by,

$$\pi_i(k+1) = \lambda_\pi \pi_i(k) + e_i^2(k) \tag{6}$$

and the switching logic scheme defined by the following pseudo code

```
counter:=0
sigma:=1
while 1
begin
  wait_for_sample
  counter:=counter+1
  if counter >= T_D then
      if pi(sigma) > min( pi )
          sigma := argmin( pi )
          counter:=0
      endif
  endif
end
```

This switching logic scheme ensures that the controller corresponding to the minimum performance signal, π_i, is applied if T_D sampling instants have passed since the last controller commutation, otherwise the controller remains unchanged.

The interconnection of the above defined subsystems is analyzed, from the stability point of view, for linear time-invariant plants in (Morse, 1996) and (Borrelli, 1999) for continuous and discrete-time cases, respectively. A stability analysis for a case of linear time-varying discrete plant is presented in (Rato, 2002), and a robustness analysis of linear time-invariant plants is presented in (Morse, 1997).

2.2 *Composite System*

One of the key ideas to proof the stability of the controlled system is to build a composite system with state $x = [x_E'(k) \; x_C'(k)]'$, with the i-th estimator error $e_i(k)$ as the input, the control signal $u(k)$, the tracking error $e_T(k)$, and the j-th estimator error $e_j(k)$ as outputs. The stability of the closed loop system $\{K_i, P_i\}$ (for every index i) implies the detectability of the composite system through $e_i(k)$ (Hespanha, 1997).

From (2-5) through simple transformations similar to those presented in(Morse, 1996) it is possible to build the composite system with inputs $r(k)$, $e_i(k)$, and state $x(k) = [x'_E(k)\ x'_C(k)]'$,

$$x(k+1) = A_{ij}x(k) + d_i\, e_j(k) + d'_j r(k) \quad (7)$$
$$u(k) = f_{ij}x(k) + g_i\, e_j(k) + g'_j r(k) \quad (8)$$
$$y(k) = c_j x(k) - e_j(k) + r(k) \quad (9)$$
$$e_p(k) = c_{pj}x(k) +\ e_j(k) \quad (10)$$

where matrix A_{ij} is such that $A_{\sigma\sigma}$ is stable with all eigenvalues inside the circle of radius (Morse, 1996; Borrelli, 1999). Considering the process between commutations defined by

$$x_P(k+1) = A_P x_P(k) + B_P\, u(k) \quad (11)$$
$$y(k) = C_P x_P(k) \quad (12)$$

which is assumed detectable and stabilizable with unobservable or uncontrollable modes with eigenvalues inside the circle of radius $\lambda_\omega < 1$. Then, by definition, there exists K such that $A_P + K\, C_P$ as all eigenvalues inside the circle of radius λ_ω. Thus, from equation (11), the following relation is yielded

$$x_P(k+1) = (A_P + KC_P)x_P(k) + B_P u - Ky \quad (13)$$

Consider also the following input-output representation of the estimator with state equations (2-3)

$$\hat{y}_{p^*} = \frac{A_0 - A}{A_0} y\ +\ \frac{B}{A_0} u \quad (14)$$

where $A_0(z) = det(A_E - zI)$. From $e_{p^*} = \hat{y}_{p^*} - y$, yields

$$e_{p^*} = -\frac{B}{A_0}\,(\delta_m)\, u - \frac{A}{A_0}\, d \quad (15)$$

which describes the estimator error as a function of the plant unmodelled dynamics. From (6), (7-10), (13), and (15) it is possible to describe the relations between $[x'_E\, x'_C]'$, $x_P(k)$ and $\pi(k)$ by the block diagram in figure 3. The main idea to verify the system stability

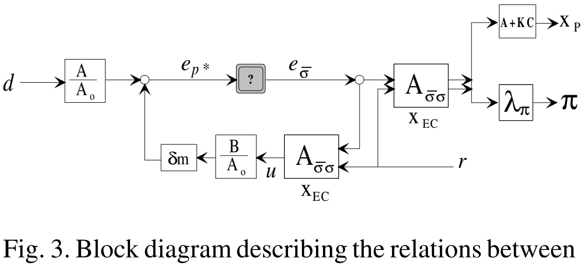

Fig. 3. Block diagram describing the relations between $x_E(k)$, $x_C(k)$, $x_P(k)$ and $\pi(k)$.

is to devise a relation between a norm of the estimator error, $\|e_{p^*}\|$, and a norm of $\|e_i\|$. If the there is a finite gain from $\|e_{p^*}\|$ to $\|e_i\|$, and the induced norm of $\|A_{\bar\sigma\sigma}\|$ is finite then applying the small gain theorem the system will be stable if $\|\delta_m\|$ is small enough, for every time period with constant plant. Further, if the overall system is exponentially uniformly stable then there will be a dwell-time period for the plant such that the overall systems with time-varying plant will be stable.

Setting indices σ (the active control index) and $\bar\sigma$ to be $\bar\sigma = \sigma(k)$ yields a stable matrix $A_{\sigma\sigma}$ for the composite system $[x'_E\, x'_C]'$, which has eigenvalues inside the circle of radius $\lambda_{CECL} = max(\lambda_C, \lambda_E, \lambda_{CL})$ (Morse, 1996; Borrelli, 1999). However there is no simple way to bound $\|e_\sigma\|$ as a function of $\|e_{p^*}\|$ since the estimator error signal e_σ can be occasionally much larger then e_{p^*}. Thus, a key step to the stability proof is the definition of norm $\|.\|$ and the signal $\bar\sigma$ such that $\|A_{\bar\sigma\sigma}\|$ as well as the relation between $\|e_{p^*}\|$ and $\|e_{\bar\sigma}\|$ have a finite induced norm gain.

3. MAIN RESULT

Throughout this paper the following assumptions are made.

Assumption 1 *The plant dynamic behavior associated to index p^* varies at time instants $k^*_1, k^*_2, k^*_3, ...$, so the process dynamic is kept constant in the intervals $k^*_i \le k < k^*_{i+1}$.*

Assumption 2 *The controller associated to index σ varies at time instants $k_1, k_2, k_3, ...$, so the controller is is kept constant in the intervals $k_i \le k < k_{i+1}$.*

A key step to the stability proof of of the overall system is the definition of the indices i and j of the composite system. Let $j(k) = \sigma(k)$ and $i(k) = \bar\sigma(k)$ where $\bar\sigma$ is defined as follows. For each interval $k_i \le k < k_{i+1}$ let

$$\overline{\sigma}(k) = \begin{cases} \sigma(k) & if\ \pi_{\sigma(k_i)}(k_i) \le \pi_{p^*}(k^*_b)\, \lambda^{k_i - k^*_b} \\ p^* & if\ \pi_{\sigma(k_i)}(k_i) > \pi_{p^*}(k^*_b)\, \lambda^{k_i - k^*_b} \end{cases} \quad (16)$$

and for the last interval $k_i \le k < k^*_{l+1}$ within $[k^*_l, k^*_{l+1} - 1]$, define

$$\overline{\sigma}(k) = \begin{cases} \sigma(k) & if\ k^*_b - k_u \ge T_D \\ p^* & if\ k^*_b - k_u < T_D \end{cases} \quad (17)$$

The intuition to this definition comes from the following lemma.

Lemma 1 *For any two indices i and j and according to the performance signals definition (6)*

$$\pi_i(k^*_a) \ge \pi_j(k^*_b)\, \lambda_\pi^{k^*_a - k^*_b} \implies \pi_i(k) \ge \pi_j(k) \quad (18)$$

for any k, such that $k^*_l \le k < k^*_{l+1}$

From Lemma 1 it can be seen that function $\pi_{p^*}(k^*_b)\, \lambda_\pi^{k - k^*_b}$ behaves like a "temporary rejection" frontier line. Thus, whenever a performance signal goes above that line it will never go below it as long as the process is kept constant.

The definition of $\overline{\sigma}$ is such that $\overline{\sigma} = p^*$ in a certain interval $k_i \leq k < k_{i+1}$ if model with index $\sigma(i)$ has been "temporarily rejected", otherwise $\overline{\sigma} = \sigma$. It is now clear the purpose of the definition of $\bar{\sigma}$. As the number of rejections is not larger than the number of models, which is finite, then the number of time intervals in which $\overline{\sigma} \neq \sigma$ is limited.

From this fact it is proven that the composite system (which is time-varying) with state matrix $A_{\overline{\sigma}\sigma}$ is stable. This is stated in the following key lemmas.

Lemma 2 *Consider the function $\psi(k)$ such as $\psi(k) = 1$ if $\sigma = p^*$ and $\psi(k) = 0$ otherwise. Then*

$$\sum_{j=k_i^*}^{k_{i+1}^*-1} \psi(k) \leq T_D\,(n-1) \tag{19}$$

where T_D is the controller dwell-time imposed by the switching logic scheme.

Lemma 2 states that the period of time such that $\overline{\sigma} \neq \sigma$, within an interval $k_l^* \leq k < k_{l+1}^*$ is limited, and is a basis for following Lemma.

Lemma 3 *Consider $\bar{\sigma}$ defined by (16-17) then the composite system with state matrix $A_{\bar{\sigma}\sigma}$ is uniformly exponentially stable with stability margin*[3] *$\lambda_{CECL} = max(\lambda_C, \lambda_E, \lambda_{CL})$ within any interval $k_l^* \leq k < k_{l+1}^*$*

The proof of the above Lemmas is omitted in this paper, see (Rato, 2002). Another key step is the definition of the norm:

Definition 1 *For any discrete signal $x(k)$ let $\|x(k)\|_{\lambda(k_0,k)}$ denote the exponentially weighted norm*

$$\|x(k)\|_{\lambda(k_0,k)} = \sqrt{\sum_{j=k_0}^{k-1} \lambda^{2(k_0-j-1)} x^2(j)} \tag{20}$$

The utility of this norm comes from the following identity, easily derived from (6) and Definition 1,

$$\pi_i(k)\lambda_\pi^{-k} = \pi_i(k_0)\lambda_\pi^{-k_0} + \lambda_\pi^{-k_0}\|e(k)\|^2_{\lambda_\pi^{1/2}(k_0,k)} \tag{21}$$

Thus, the definition of $\bar{\sigma}$, apart from the initial conditions, can be described comparing $\|e_{p^*}\|^2_{\lambda(k_0,k)}$ and $\|e_\sigma\|^2_{\lambda(k_0,k)}$.

To determine the relation between $\|e_{p^*}\|^2_{\lambda(k_0,k)}$ and $\|e_{\bar{\sigma}}\|^2_{\lambda(k_0,k)}$ consider the set of functions $\phi_i(k)$ defining the instants such that the i-th controller is active. Thus, let $\phi_i(k)$ be defined by

$$\phi_i(k) = \begin{cases} 1 & if \quad \bar{\sigma}(k) = i \\ 0 & c.c. \end{cases} \tag{22}$$

[3] A linear systems with state x is exponentially uniformly stable with stability margin λ, if there exist a finite γ and $0 \leq \lambda < 1$ such that $\|x(k)\| \leq \gamma\lambda^{k-k_0}\|x(k_0)\| + b$, where $b \neq 0$ if there is a bias.

Figure 4 shows an example of such functions. Define signal $e_{\bar{\sigma}(k)}(k)$ as a function of the prediction errors

$$e_{\bar{\sigma}(k)}(k) = \sum_{i\in\mathcal{P}} \phi_i(k)e_i(k) \tag{23}$$

According to the weighted norm definition

$$\|e_{\bar{\sigma}(k)}(k)\|^2_{\lambda(k_0,k)} = \sum_{j=k_0}^{k-1} \lambda^{k_0-j-1} e^2_{\bar{\sigma}(j)}(j) +$$
$$= \sum_{j=k_0}^{k-1} \lambda^{k_0-j-1}\left(\sum_{i\in\mathcal{P}} \phi_i(j)e_i(j)\right)^2 \tag{24}$$

For each instant k there exists only one index $\bar{\sigma}$ such that function $\phi_{\bar{\sigma}} \neq 0$

$$\|e_{\bar{\sigma}(k)}(k)\|^2_{\lambda(k_0,k)} = \sum_{j=k_0}^{k-1} \lambda^{k_0-j-1} \sum_{i\in\mathcal{P}} \phi_i^2(j)e_i^2(j) \tag{25}$$

and as $\phi_i^2(k) = \phi_i(k)$

$$\|e_{\bar{\sigma}(k)}(k)\|^2_{\lambda(k_0,k)} = \sum_{j=k_0}^{k-1} \lambda^{k_0-j-1} \sum_{i\in\mathcal{P}} \phi_i(j)e_i^2(j) +$$
$$= \sum_{j=k_0}^{k-1}\sum_{i\in\mathcal{P}} \lambda^{k_0-j-1}\phi_i(j)e_i^2(j) \tag{26}$$

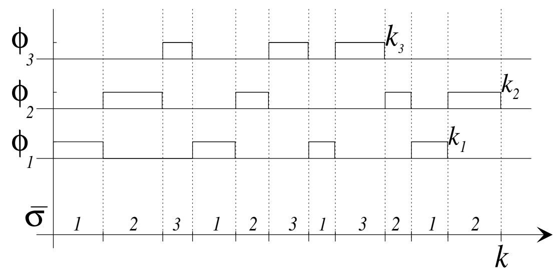

Fig. 4. Division of time instants according to index $\bar{\sigma}$.

The definition of ϕ_i, implies that for every instant there is one and only one index i such that $\phi_i \neq 0$. Thus it can be easily seen that the order of the sums can be interchanged

$$\|e_{\bar{\sigma}(k)}(k)\|^2_{\lambda(k_0,k)} = \sum_{i\in\mathcal{P}}\sum_{j=k_0}^{k-1} \lambda^{k_0-j-1}\phi_i(j)e_i^2(j) \tag{27}$$

For each $i \in \mathcal{P}$ let $\bar{k}_i \leq k$ be the largest time instant such that $\phi_i \neq 0$. It should be noted that $\bar{k}_i$ coincides with a commutation instant of the controller, defined by index σ, except to the active index at instant k for which $\bar{k}_{\bar{\sigma}(k)} = \bar{k}_{\sigma(k)} = k$.

For the terms corresponding to i, $\phi_i(j) = 0$ for each $j > \bar{k}_i$, thus

$$\|e_{\bar{\sigma}(k)}(k)\|^2_{\lambda(k_0,k)} = \sum_{i\in\mathcal{P}}\sum_{j=k_0}^{\bar{k}_i-1} \lambda^{k_0-j-1}\phi_i e_i^2(j) \tag{28}$$

As all terms are positive and $0 \leq \phi_i \leq 1$

$$\|e_{\bar{\sigma}(k)}(k)\|^2_{\lambda(k_0,k)} \leq \sum_{i\in\mathcal{P}}\sum_{j=k_0}^{\bar{k}_i-1} \lambda^{k_0-j-1} e_i^2(j) =$$

$$= \sum_{i\in\mathcal{P}} \|e_i\|^2_{\lambda(k_0,\bar{k}_i)} \quad (29)$$

where $\bar{k}_i < k$. By definition of $\bar{\sigma}$

$$\pi_{\bar{\sigma}(k')}(k')\lambda_\pi^{-k'} \leq \pi_{p^*}(k)\lambda_\pi^{-k} \quad for \quad k' < k \quad (30)$$

Applying (21) and choosing λ such that $\lambda_\pi^{1/2} < \lambda < 1$

$$\|e_{\bar{\sigma}(k')}(k')\|^2_{\lambda(k_0,k')} \leq \|e_{p^*}(k)\|^2_{\lambda(k_0,k)} + \\ + \left(\pi_{p^*}(k_0) - \pi_{\bar{\sigma}(k')}(k_0)\right) \quad (31)$$

so if $k' = \bar{k}_i$

$$\|e_{\bar{\sigma}(\bar{k}_i)}(\bar{k}_i)\|^2_{\lambda(k_0,\bar{k}_i)} \leq \|e_{p^*}(k)\|^2_{\lambda(k_0,k)} + \\ + \left(\pi_{p^*}(k_0) - \pi_{\bar{\sigma}(\bar{k}_i)}(k_0)\right) \quad (32)$$

It should be noted that when $k = \bar{k}_i$ index $\bar{\sigma} = i$, thus $\bar{\sigma}(\bar{k}_i) = i$.

$$\|e_i(\bar{k}_i)\|^2_{\lambda(k_0,\bar{k}_i)} \leq \|e_{p^*}(k)\|^2_{\lambda(k_0,k)} + (\pi_{p^*}(k_0) - \pi_i(k_0)) \quad (33)$$

Substituting (29),

$$\|e_{\bar{\sigma}(k)}(k)\|^2_{\lambda(k_0,k)} \leq \sum_{i\in\mathcal{P}} \|e_{p^*}\|^2_{\lambda(k_0,k)} + \\ + \sum_{i\in\mathcal{P}} (\pi_{p^*}(k_0) - \pi_i(k_0))) \quad (34)$$

Considering the number of controllers/models to be n, for $k \leq k'$, yields

$$\|e_{\bar{\sigma}(k)}(k)\|^2_{\lambda(k_0,k)} \leq n\|e_{p^*}\|^2_{\lambda(k_0,k)} + n\, C_\pi\, \Pi(k_0) \quad (35)$$

$$\|e_{\bar{\sigma}(k)}(k)\|_{\lambda(k_0,k)} \leq \sqrt{n}\|e_{p^*}\|_{\lambda(k_0,k)} + \sqrt{nC_\pi|\Pi(k_0)|} \quad (36)$$

where $C_\pi = \|C_{\pi p^* i}\|$, and this is such that $\pi_{p^*}(k_0) - \pi_i(k_0) = C_{\pi p^* i}\Pi(k_0)$ thus $C_{\pi p^* i}$ is a vector constituted by zeros except element with index i which is "-1" and element with index p^* with value "1", which euclidean norm is $\|C_{\pi p^* i}\| = \sqrt{2}$. In the particular case of $i = p^*$, vector $C_{\pi p^* i}$ is identically zero. This result is stated in the following Lemma.

Lemma 4 *Consider the overall system within an interval such that the process dynamics is constant, defined by equations (6), (7-10), (13), (15) the switching signal σ according to the dwell-time switching scheme, $\bar{\sigma}$ defined by (16-17), and the weighted norm according to Definition 1, then*

$$\|e_{\bar{\sigma}(k)}(k)\|_{\lambda(k_0,k)} \leq \sqrt{n}\|e_{p^*}\|_{\lambda(k_0,k)} + \sqrt{nC_\pi|\Pi(k_0)|}$$

From Definition 1 it is easily proved that a linear system has finite weighted norm if it is exponentially uniformly stable with stability margin λ. Thus, it is possible to apply the small gain theorem to the loop depicted in figure 3.

Consider the induced norm of the composite system with state $[x'_C\; x'_E]'$, input $e_{\bar{\sigma}}$ and output u, denoted by $\|\{A_{\bar{\sigma}\sigma}, e_{\bar{\sigma}} \to u\}\|_\lambda$. Applying the small theorem gain it is then possible to guarantee the system stability if

$$\|\delta_m\|_\lambda < \frac{1}{\sqrt{n}\|\{A_{\bar{\sigma}\sigma}, e_{\bar{\sigma}} \to u\}\|_\lambda \,\|\frac{B}{A_0}\|_\lambda \,\|\delta_m\|_\lambda} \quad (37)$$

This result and the extension to the case of time varying plants are stated in the following theorems whose detailed proof are omitted in this paper.

Theorem 1 *Consider the overall system within an interval such that the process dynamics is constant, defined by equations (6), (7-10), (13), the switching signal σ with a dwell-time T_D, and $\bar{\sigma}$ defined by (16-17), then there is $0 < \lambda < 1$, dwell-time $T_D > 0$, and $\delta > 0$ such that, if $\|\delta_m\|_\lambda < \delta$, with*

$$\delta = \frac{1}{\sqrt{n}\|\{A_{\bar{\sigma}\sigma}, e_{\bar{\sigma}} \to u\}\|_\lambda \,\|\frac{B}{A_0}\|_\lambda \,\|\delta_m\|_\lambda} \quad (38)$$

the overall system with state $[x'_C\; x'_E]'$, x_P, and π is uniformly exponentially stable with stability margin

$$\lambda_{\pi\omega CECL} = max(\lambda_\pi^{1/2}, \lambda_\omega, \lambda_C, \lambda_E, \lambda_{CL})$$

If a dwell-time condition is imposed to a set of uniformly exponentially stable systems then the commutation among those systems will be exponentially stable. This conclusion leads to Theorem 2. The proof of Theorem 2 follows the proof of Lemma 4 in (Morse, 1996), extended to the case when the system commutes among time-varying subsystems.

Theorem 2 *Consider the overall system defined as in Theorem 1, if the process commutes among the admissible models assuring a minimum time between commutations T_{Dp}, then the overall time-varying system is exponentially stable.*

4. SIMULATION RESULTS

The plant considered in this simulation example is a double cart coupled by a link with a varying stiffness parameter (Mosca, 1999), in figure 5. The control problem is to position cart n. 2 applying a force at cart n. 1. The controllers and models used in the following simulations are those defined in (Mosca, 1999), which were designed according to a weighted H_∞ mixed-sensitivity method (Kwakernaak, 1993). The discretization of the nominal models $\frac{B_i}{A_i}$ and the controllers $\frac{S_i}{R_i}$ was done assuming an ideal zero order hold with sampling time $T_s = 0.1\, s$. Nominal model 1 correspond to a stiffness $\gamma_1 = 0.3$ and nominal model 2 corresponds to $\gamma_2 = 1.0$.

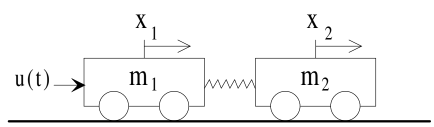

Fig. 5. Plant - coupled double cart with variable stiffness.

In this simulation examples the models are calculated from the nominal ones admitting a 2.5% change in the parameters. Nevertheless, the double pole remains unaltered as this is a dynamic behavior that is intrinsic to physics of the plant. The considered models are:

$$A_1'(q) = q^4 - 4.004q^3 + 6.018q^2 - 4.024q + 1.01$$
$$B_1'(q) = 1.27e - 6q^3 + 1.354e - 5q^2 + 1.394e$$
$$-5q + 1.23e - 6$$
$$A_2'(q) = q^4 - 4.004q^3 + 6.0319q^2 - 4.0518q + 1.0239$$
$$B_2'(q) = 4.164e - 6q^3 + 4.575e - 5q^2 + 4.575e$$
$$-5q + 4.164e - 6$$

The simulation were performed with dither noise with power $\eta^2 = 10^{-9}$ and the plant commutes among model 1 and 2 every $100\,s$. Figures 6 and 8 show

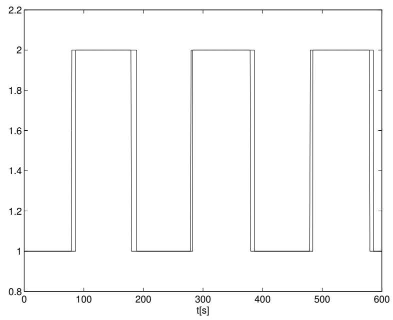

Fig. 6. Active plant index and supervisor decision σ, in the presence of unmodelled dynamics.

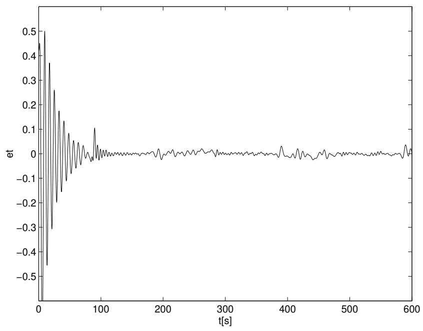

Fig. 7. Tracking error e_T, in the presence of unmodelled dynamics.

that the system remains stable in the presence of unmodelled dynamics. Due the presence of unmodelled

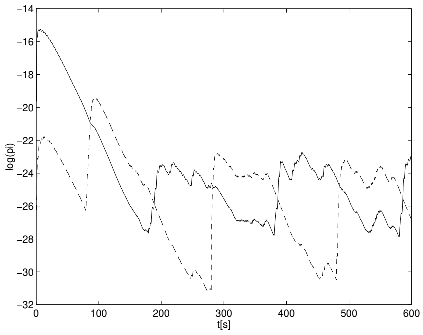

Fig. 8. Performance signals logarithm $log(\pi_i)$.

dynamics, the estimator errors don not decay exponentially to zero. This is shown in 8, which represents the performance signals logarithm. The values of the performance signals can be interpreted as a distance between the actual plant and each of the nominal models.

5. CONCLUSIONS

A switching control algorithm applied to linear time-varying plants in the presence of unmodelled dynamics as been analyzed. The stability proof has been outlined and conditions have been posed to ensure stability of the controlled system. The conditions to ensure stability have been defined as a "dwell-time" on the plant dynamics as well as a limited weighted norm on the multiplicative description of the unmodelled dynamics. Simulation examples have been presented showing the switching control behavior in the presence of unmodelled dynamics.

6. REFERENCES

Borrelli, D., A. Stephen Morse, Edoardo Mosca, "Discrete-Time Supervisory Control of Families of Two-Degrees-of-Freedom Linear Set-Point Controllers", *IEEE Transactions on Automatic Control*, **Vol. 44**, 178-181.(1999).

Hespanha, J., A. S. Morse, "Certainty Equivalence Implies Detectability", *Systems and Control Letters*, August (1997)

Kwakernaak, H. "Robust Control and H_∞ Optimization - Tutorial Paper", *Automatica*, **vol. 29**, 2, pág. 255-273, (1993).

Lemos, J. M., L. M. Rato, J. S. Marques, "Switching Reconfigurable Control Based on Hidden Markov Models", *ECC99, European Control Conference, Karlsruhe 99* , (1999)

Morse, A. S., "Control Using Logic-Based Switching" in "Trends in Control- an European Perspective", A. Isidori (editor), *Springer-Verlag*, (1995).

Morse, A. S. "Supervisory Control of Families of Linear Set-Point Controllers - Part 1: Exact matching", *IEEE Trans. Automat. Contr.*, **vol. 41**, 1413-1431, (1996).

Morse, A. S. "Supervisory control of Families of Linear Set-Point Controllers - Part 2: Robustness", *IEEE Trans. Automat. Contr.*, **vol. 42**, 1500-1515, (1997).

Mosca, E., T.Agnoloni, "Closed-Loop Performance Inference for Controller Selection in Switching Supervisory Control", *ECC99, European Control Conference, Karlsruhe*, (1999)

Rato, L. M., D. Borrelli, E. Mosca, J. M. Lemos, P. Balsa, "MUSMAR based switching control of a solar collector field" *ECC97, European Control Conference, Brussels 97* , (1997)

Rato, L. M., J. M. Lemos, "Switching Control of Time-Varying Discrete Plants" *(Submitted), MED02, Mediterranean Control Conference, Lisbon 02* , (2002)