MULTIRATE MUSMAR CASCADE CONTROL OF A DISTRIBUTED COLLECTOR SOLAR FIELD

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Abstract: Experimental results are presented on the application of a cascade control structure to a distributed collector solar field. Both loops of the cascade use MUSMAR, a predictive adaptive control algorithm. The difference in the dominant time constants of the inner (faster) and outer (slower) control loops is explored by employing different sampling rates in each of them. An analysis is presented characterising the possible convergence points of the controllers and relating them to steady-state LQ stochastic control.

1 Introduction

Cascade control is a classical structure in process control [1]. It explores the situation in which the system to be controlled can be split in two series subsystems P_1 and P_2 (fig.1) with the dominant time constant of P_1 being much faster than the dominant time constant of P_2. The two controllers C_1 and C_2 are connected such that the manipulated variable of C_2, u_2, is the setpoint of C_1, thus forming two nested loops. Further, in a computer control framework, again exploring the difference between the dominant poles of P_1 and P_2, these subsystems are sampled at different rates h_1 and h_2, with

$$h_2 > h_1$$

As is well known, the main advantage of the cascade structure consists in the fast rejection of the disturbances entering the inner loop, without letting them affect the outer loop, where they would take much longer to be rejected. Common examples of cascade control are provided, e. g. in large scale thermoelectric power units by the regulation of drum boiler level using so-called "three element control" and by regulation of superheated steam temperature. In [2] one example is provided of using predictive control in a cascade structure.

In this paper, experimental results are presented on the application of a cascade control structure to the oil outlet temperature in a distributed collector solar field. Both loops of the cascade employ MUSMAR [3], a predictive adaptive control algorithm for which there is evidence of robustness against plant unmodelled dynamics [4]. An analysis based on singular perturbation methods is presented in order to extend the results of [4] to multirate cascade control.

Figure 1: Multirate cascade control structure.

There is a rich literature on the control of the inner loop of the plant considered, of which [5]-[8] are significative examples. The use of MUSMAR together with the above mentioned multirate sampling scheme provides an alternative way of controlling this plant which is particularly effective in the outer loop where a long, time varying, pure delay is present.

The paper is organised as follows: In section 2 the plant to control is briefly described and mention is made of the main difficulties faced when controlling it, together with a discussion on the suitability of employing cascade control. In section 3 the algorithm used, which also embodies feedback from accessible disturbances is presented. In section 4 an analysis characterising possible convergence points of the controllers is made. Section 5 describes experimental results obtained by runing the algorithm in the distributed collector solar field and section 6 draws conclusions.

2 Plant description

The plant to be controlled consists in a distributed solar collector field (manufactured from ACUREX) which is part of Plataforma Solar de Almeria, located in the south of Spain. Since complete descriptions of the field are available in the literature [5, 6, 8], this paper only refers the details more pertinent to the work at hand. The field is

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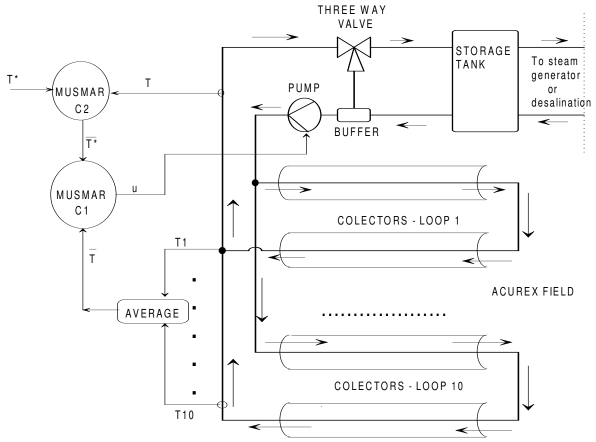

Figure 2: Schematic view of the solar field with cascade control.

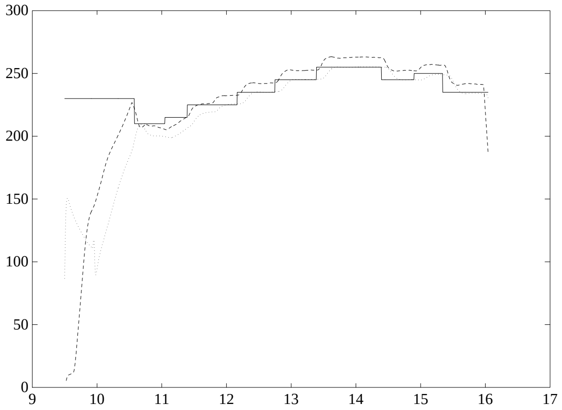

Figure 3: The reference to follow (solid), the average temperature at the outlet of the collector loops (dashed) and the temperature at the inlet to the storage tank (dotted).

made of 480 cilindrical collectors of parabolic type, which concentrate the solar radiation in a metallic tube located along their focus. The metallic tube contains a fluid (oil) which serves as an energy storage medium. The collectors are organized in 10 parallel loops placed along an east-west axis and are provided of a sun elevation tracking mechanism. The oil to be heated is extracted from the bottom of a storage tank and is pumped through the collector loops. At the other end, the outlet of the loops (fig. 2) is collected in a passive pipe and brought back to the storage tank, where it enters at the top. The oil pump is provided with a local controller so that it can be assumed that oil flow is the manipulated variable, ranging from $2\ell/s$ to $9\ell/s$. The main process variable is the temperature of the oil entering the storage tank (y_2 in fig. 1). In a cascade control framework, the average of the temperatures at the outlet of the loops is taken as the intermediate variable (y_1 in fig. 1). In this way, the dynamics is decomposed in two parts: The dynamics of the collector loops, which relate the oil flow (u_1 in fig. 1 to y_1; and the dynamics of the pipe connecting the outlet of the loops with the inlet of the storage tank, which relates the temperatures in both points. The former is denoted by P_1 in fig.1 and corresponds to the faster time constant (with a value of about 2 minutes to 3 minutes). The latter is denoted by P_2 in fig.2 and corresponds to the slower time constant (in this case a pure delay of about 9 minutes in series with a time constant of about 2 minutes). It is remarked that the above values of the time constants are merely indicative since they vary according to the operating conditions, namelly oil flow. The main disturbances enter the system in P_1 and correspond to the variations in solar radiation and loop inlet oil temperature. The disturbances entering through P_2 correspond to losses in the pipe. In case a cascade structure would not be used in controlling the temperature at the inlet to the storage tank, the long delay in pipe dynamics would lead to a poor tuning for disturbance rejection. With cascade control, the problems of rejecting disturbances in P_1 and P_2 are split apart.

It is also remarked that the plant at hand presents distributed parameter dynamics, in particular high frequency ressonant peaks, which vary with the operating point. Thus, the controllers used for C_1 and C_2 should be such that they are able to cope with unmodelled dynamics and uncertain pure delay.

3 Control algorithm

Taking into account the difficulties mentioned, the approach followed consists in the use of the MUSMAR controller [3], including feedback from accessible disturbances (solar radiation in this case). For each of the controllers C_i, $i = 1, 2$, the algorithm reads as follows:

MUSMAR algorithm

At the begining of each sampling interval, recursivelly perform the following steps, for $i = 1, 2$:

1.Sample P_i output at time t_i, $y_i(t_i)$ and compute the error by

$$\tilde{y}_i(t_i) = y_i(t_i) - y_i^*(t_i) \tag{1}$$

where

$$y_1^* = u_2 \tag{2}$$

and y_2^* denotes the setpoint to be tracked by the oil temperature at the inlet to the storage tank.

2.Using Recursive Least Squares, update the estimates of the parameters in the following sets of predictive models:

$$\tilde{y}_i(t_i + j) = \theta_{j,i} u_i(t_i) + \psi'_{j,i} s_i(t_i) \tag{3}$$

$$u_i(t_i + j - 1) = \mu_{j-1,i} u_i(t_i) + \phi'_{j-1,i} s_i(t_i) \tag{4}$$

$$j = 1, \ldots, T_i$$

$$i = 1,\, 2$$

where $s_i(t_i)$ is a sufficient statistic for computing the control in C_i, hereafter referred as the pseudostate, given by

$$\begin{aligned} s_1(t_1) = [&y_1(t_1)\, \ldots\, y_1(t_1 - n_1 + 1) \\ &u_1(t_1 - 1)\, \ldots\, u_1(t_1 - m_1)\, y_1^*(t_1)\, I(t_1)]' \end{aligned} \tag{5}$$

$$\begin{aligned} s_2(t_2) = [&y_2(t_2)\, \ldots\, y_2(t_2 - n_2 + 1) \\ &u_2(t_2 - 1)\, \ldots\, u_2(t_2 - m_2)\, y_2^*(t_1)]' \end{aligned} \tag{6}$$

and I is the solar radiation (accessible disturbance).

3.Apply to the plant the control given by

$$u_i(t_i) = F_i' s_i(t_i) + \eta_i(t_i) \tag{7}$$

where η_i is a white dither noise of small amplitude, such that η_1 and η_2 are independent and F_i is the vector of controller gains of C_i, computed from the estimates of the corresponding predictive models by

$$F_i = -\frac{1}{\alpha_i} \sum_{j=1}^{T_i} \theta_{j,i}\psi_{j,i} + \rho_i \sum_{j=1}^{T_i - 1} \mu_{j,i}\phi_{j,i} \tag{8}$$

with the normalization factor α_i given by

$$\alpha_i = \sum_{j=1}^{T_i} \theta_{j,i}^2 + \rho_i(1 + \sum_{j=1}^{T_i - 1} \mu_{j,i}^2) \tag{9}$$

4 Algorithm convergence points

In this section an analysis is performed for characterizing the possible convergence points of the controller gains in the multirate cascade adaptive control structure of fig. 1. The analysis relies on a combination of known results for the possible convergence points for MUSMAR [4], based on the ODE method for analysing stochastic algorithms [9], with singular perturbation methods for differential equations [10].

Assume both plants P_1 and P_2 be described by an ARMAX model

$$A_i(q)y_i(t_i) = B_i(q)u_i(t_i) + C_i(q)e_i(t_i) \;\; i = 1,\, 2 \tag{10}$$

in which $A_i(q)$, $B_i(q)$ and $C_i(q)$ are polynomials in the forward shift operator q such that $A_i(q)$ and $B_i(q)$, and $A_i(q)$ and $C_i(q)$ have no unstable common factors and $\{e_i(t)\}$, $i = 1, 2$, are zero mean white noise sequences of identically distributed random variables, independent among each other. The variable t_i, $i = 1, 2$, denotes discrete time in the time scale corresponding to each of the loops.

Each of the MUSMAR controllers C_1 and C_2 minimizes the multistep quadratic cost

$$J_i = E\left[\sum_{k=1}^{T} \tilde{y}^2(t + k) + \rho u^2(t + k - 1) | I^t\right] \tag{11}$$

where $E[\cdot|I^t]$ stands for the mean conditioned on the information available up to time t, I^t.

Since J_1 is associated to the inner loop, it depends only on the gains F_1 of C_1. On the other way, C_2 sees as the dynamic of the plant to control, P_2 in series with the closed-loop dynamics of the inner loop. Thus, J_2 is a function of both F_1 and F_2.

According to [4] the ODEs describing the average evolution of the controller gains of C_1 and C_2, in the framework of [9] are

$$\dot{F}_1 = -\gamma \frac{1}{\alpha_1} R_{s_1}^{-1} \nabla J_1(F_1) \tag{12}$$

$$\dot{F}_2 = -\frac{1}{\alpha_2} R_{s_2}^{-1} \nabla J_2(F_1, F_2) \tag{13}$$

where the $\alpha_i > 0$, $i = 1, 2$, are defined by (9) and R_{s_i}, $i = 1, 2$, are the strictly positive definite covariance matrix of s_i, $i = 1, 2$. The dot denotes derivative with respect to equivalent continuous time τ [9] in the slow time scale corresponding to the outer-loop controller. The parameter γ in (12) defines the time scale separation in the adaptation speed, between the inner and the outer loop controllers. This parameter may be yielded either by the definition of the equivalent time as given in [10] or by including a gain in the RLS identifier.

The equations (12) and (13) are now analysed by singular perturbations in the situation in which

$$\gamma \to \infty \tag{14}$$

and assuming that the horizon T is large enough such that both J_1 and J_2 closely approximate the steady-state quadratic cost.

For that sake rewrite (12) as

$$\varepsilon \dot{F}_1 = -\frac{1}{\alpha_1} R_{s_1}^{-1} \nabla J_1(F_1) \tag{15}$$

with

$$\varepsilon \overset{\text{def}}{=} \frac{1}{\gamma} \tag{16}$$

and consider the situation in which $\varepsilon \to 0$.

Assumption 1

The algebraic equation obtained by making $\varepsilon = 0$ in (15):

$$\frac{1}{\alpha_1} R_{s_1}^{-1} \nabla J_1(\bar{F}_1) = 0 \tag{17}$$

has a finite number of isolated roots.

□

The verification of Assumption 1 depends on the stucture imposed to the controller, with respect to the plant.

In most cases of interest this assumption is met, see e.g. the examples in [4]. Since $\alpha_1 > 0$ and $R_{s_1}^{-1} > 0$ the roots of (17) correspond to either the minima or the maxima of J_1. Hereafter, $\bar{F}_1$ denotes one of the roots of (17).

Define the boundary layer correction by

$$\hat{F}_1 \stackrel{\mathrm{def}}{=} F_1 - \bar{F}_1 \tag{18}$$

with F_1 given by the solution of (15), with ε small but not necessarily zero, and consider the boundary layer system

$$\frac{d\hat{F}_1}{d\tau_1} = -\frac{1}{\alpha_1} R_{s_1}^{-1} \nabla J_1(\hat{F}_1 + \bar{F}_1) \tag{19}$$

in the fast time scale

$$\tau_1 = \frac{\tau}{\varepsilon} \tag{20}$$

Assumption 2

The equilibrium $F_1(\tau_1) = 0$ of the boundary layer system (19) is assimptotically stable and $F_1(0) - \bar{F}_1$ belongs to its domain of attraction.

□

Assumption 3

The eigenvalues of

$$\frac{\delta}{\delta F_1}\left[-\frac{1}{\alpha_1} R_{s_1}^{-1} \nabla J_1(\hat{F}_1 + \bar{F}_1)\right] \tag{21}$$

evaluated for $F_1 = \bar{F}_1$ have real parts smaller than a fixed negative number.

□

Assumption 2 and 3 hold for values of $\bar{F}_1$ which correspond to the local minima of J_1. This is readily proved by considering as Lyapunov function associated to (19) the cost $J_1(F_1)$.

Consider now the reduced system obtained by replacing F_1 in (13) with $\bar{F}_1$ corresponding to a minima of J_1:

$$\dot{F}_2 = -\frac{1}{\alpha_2} R_{s_2}^{-1} \nabla J_2(\bar{F}_1, F_2) \tag{22}$$

the equilibria of (22) corresponding to the minima of J_2 are exponentially stable. By taking this into consideration, together with the conditions for assumptions 1, 2 and 3 to hold, and by use of Corolary 2.3 of chapter 7.2 of [10], the following theorem is proved:

Theorem 1

Let G_{C_1} be the dynamics corresponding to the closed inner loop when the gains of C_1 have converged.

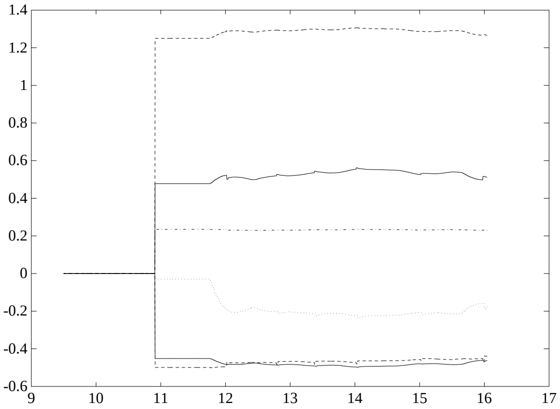

Figure 4: Controller gains – external control loop.

For T large enough, there is a separation in the sampling rates of the MUSMAR cascade controllers in fig. 1 such that their only possible convergence gains are closely approximations to the optimal steady-state stochastic LQ controller tuned for the plant P_1 in the inner loop, and tuned for the dynamics made up by the series of G_{C_1} and P_2 in the outer loop.

□

5 Experimental results

Figs. 3 to 6 report experimental results obtained on 12 February 1996. In this paper, temperature is expressed in [^oC], radiation in [W/m^2] and flow in [ℓ/s].

Both T_1 and T_2 were made equal to 15 samples. This value was found from previous experiments and simulation to be adequate. The value of T_i results from a compromise between approaching the steady-state Linear Quadratic stochastic controller (for which sake T_i should be as big as possible) and the precision in the estimate of the predictors (which decreases with T_i).

The sampling intervals were $h_1 = 15\,sec$, $h_2 = 60\,sec$. When h_2 increases, the pure delay in P_2, measured in number of samples, decreases. Again, a compromise has to be found because a too long sampling interval leads to an excessive delay in the reaction to disturbances.

The orders were set equal in both controllers and given by $n_i = 3$, $m_i = 2$. Again these values were found by systematic simulations and experiments.

The control weight ρ_i is set equal to 0.001 in both controllers. The variance of the dither noise was 0.01 in both cases.

A crucial issue in smooth startup is to start both controllers with fixed gains, obtained from previous gains or simulations. Then, after a while, the inner loop controller

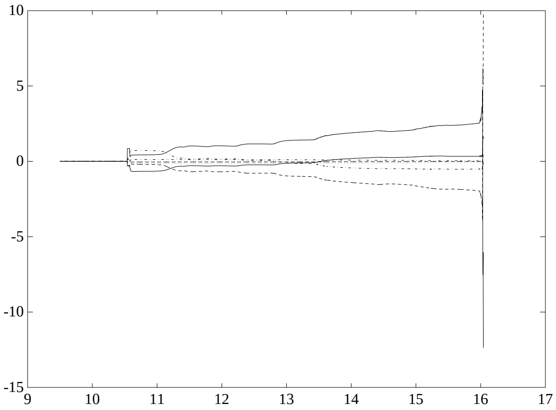

Figure 5: Controller gains – internal control loop.

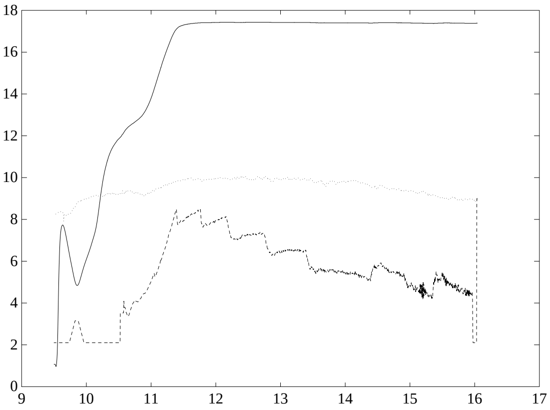

Figure 6: Field inlet temperature divided by 10 (solid), oil flow (dashed) and radiation divided by 100 (dotted).

is turned to adaptive mode and only after this has converged the outer loop is also turned to adaptive mode.

The pure delay is apparent in fig. 3, where the heat loss in the pipe connecting the outlet of the collector loops to the storage tank is also seen. The outer loop controller adjusts the setpoint of the inner loop, such that y_1 is high enough to compensate for the thermal losses in the pipe and let the temperature at the inlet of the tank be equal to the setpoint.

6 Conclusion

This paper presents experimental results on the use of a cascade structure for the control of temperature in a distributed collector solar field. Predictive adaptive controllers are used, working at different sampling rates. An analysis is presented characterising the possible convergence points of the controllers and relating them to steady-state LQ stochastic control. Cascade control has a classical widespread use with PID in industrial applications. Employing predictive, adaptive and multirate control allows a significative increase in its capabilities.

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